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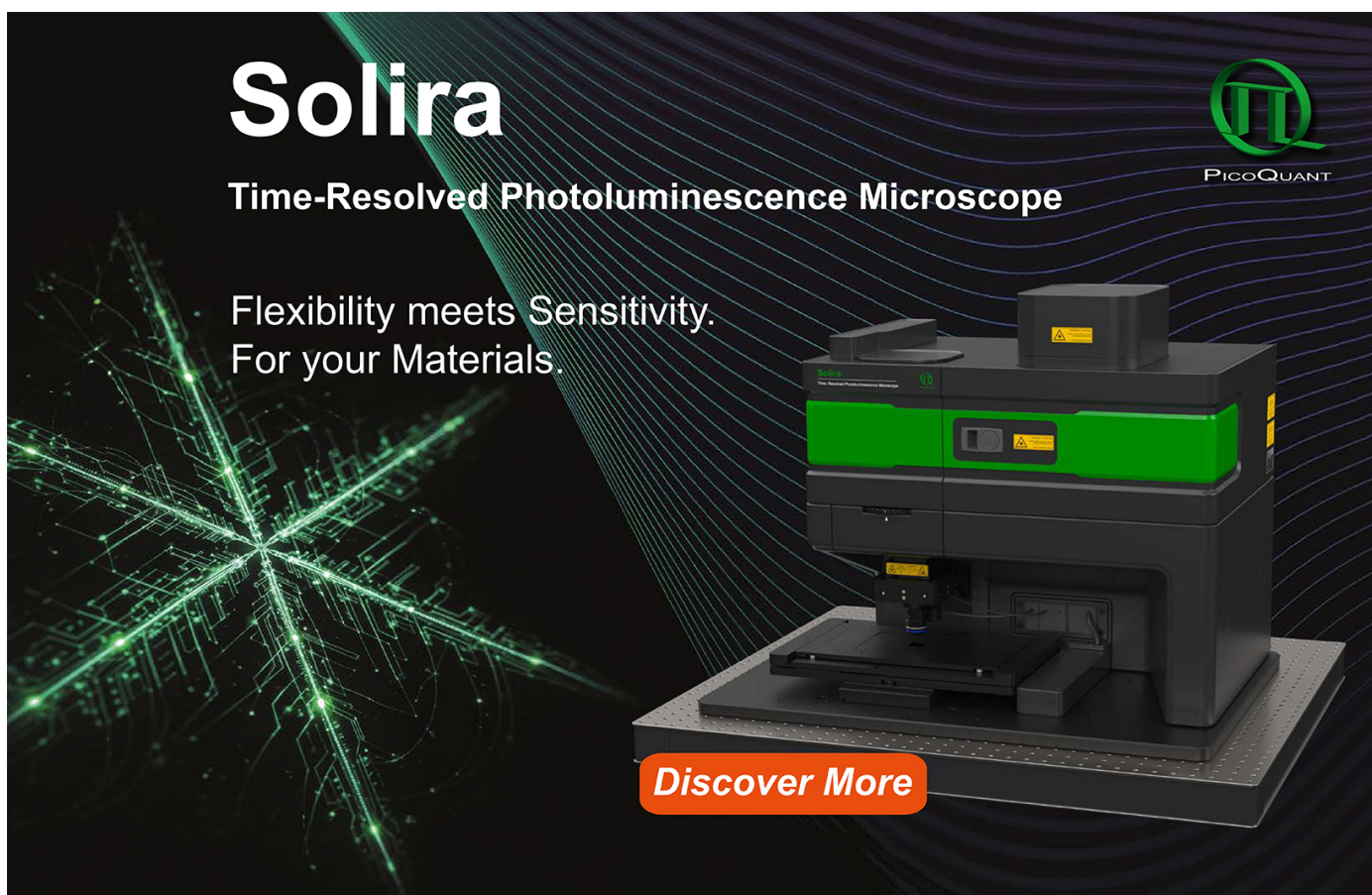
Determination of transparent thin-film refractive index using reflectivity extrema

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Determination of transparent thin-film refractive index using reflectivity extrema

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E-mail: ggt@phys.uni-sofia.bg**Keywords:** transparent films, refractive index, reflectivity extrema

Abstract

A new approach for determination the refractive index of a transparent thin film on top of an arbitrary substrate from reflectivity data taken at different layer thickness is proposed. It is shown that the layer refractive index can be found solely from minimum and maximum reflectivity values, without requiring prior knowledge of the substrate's properties. The substrate can be composed of multiple plane-parallel homogeneous layers with complex refractive indices or may exhibit an arbitrary depth-dependent refractive index profile. These results can also be applied to spectroscopic data but are limited to substrates that are homogeneous and exhibit low refractive index dispersion within the wavelength range used. The proposed theoretical approach is demonstrated by applying it to real experimental reflectivity data from interference fringes images and spectroscopic measurements taken onto a simple substrate and simulated data for a multilayer system.

1. Introduction

The refractive index is one of the most important optical parameters of a material. Measurement techniques have evolved from basic angle measurements and light deviation prism-based methods to modern tools like ellipsometry and interferometry [1]. Thin-film interference fringes, seen as minima and maxima in reflectivity, arise from changes in either layer thickness or wavelength. At a fixed wavelength, varying thickness alters the optical path, creating interference. Similarly, changing the wavelength at a fixed thickness produces the same effect.

Analyzing thin-film interference fringes is a common method to determine layer properties like refractive index and thickness. One approach uses the positions of fringe maxima and minima, but additional information is often required to separate refractive index and thickness. For instance, precise thickness measurements to extract refractive index [2], or two sets of interference patterns recorded at two different angles of incidence to find both the refractive index and the film thickness [3].

Another approach is to use the values of the power reflection coefficients or fringe maxima and/or minima values instead of their separation as input data. Examples are reflectance data to find refractive index of liquids [4], using combined transmittance and reflectance data [5] or using min/max envelope of *in situ* optical reflectance data during layer deposition [6].

The results presented here demonstrate that the refractive index of a transparent layer can be determined solely from minimum and maximum reflectivity values. The solution is simple and analytical, with main advantage that it does not require any prior knowledge of the underlying substrate properties or layer thickness. By analyzing the complex plane trajectory of amplitude reflection coefficients, exact expressions are derived for refractive index extraction from reflectivity data at any incidence angle and for both s- and p-polarizations. These formulas depend only on the reflectivity minima and maxima, with no additional system parameters needed. For variable thickness reflectivity data, this method applies to any layer/substrate configuration, including substrates with multiple homogeneous layers or

arbitrary depth-dependent refractive index profiles. Variable thickness reflectance data can be collected from Newton fringes [7], wedge interferometer Fizeau fringes [8], or real time *in situ* measurements during layer deposition [6], epitaxial growth [9], etching [10], oxidation [11], adsorption [12], spin coating [13, 14].

The results presented here can be applied also to spectroscopic data but are limited to layer on homogeneous substrate and for low refractive index dispersion within the wavelength range used.

2. System under consideration

The system under consideration consists of a transparent top layer with a real refractive index n_1 on a substrate, placed in a nonabsorbing ambient with a real refractive index n_0 . We examine a very general case for the substrate, with the only requirement being that its scattering matrix is diagonal in the p/s basis presentation [15]. This encompasses substrates with any number of plane-parallel homogeneous layers with complex refractive indices, as well as those with arbitrary depth-dependent refractive index profiles.

The complex amplitude reflection coefficients R_p and R_s for p/s polarization can be expressed by the top layer parameters and the underlying system optical response [15]

$$R_{p/s} = \frac{r_{01p/s} + R_{1xp/s}Z}{1 + r_{01p/s}R_{1xp/s}Z} \quad (1)$$

$$Z = \exp\left(4\pi i \frac{d}{\lambda} n_1 \cos(\varphi_1)\right) \quad (2)$$

$$r_{01p} = \frac{n_1 \cos(\varphi_0) - n_0 \cos(\varphi_1)}{n_1 \cos(\varphi_0) + n_0 \cos(\varphi_1)} \quad (3)$$

$$r_{01s} = \frac{n_0 \cos(\varphi_0) - n_1 \cos(\varphi_1)}{n_0 \cos(\varphi_0) + n_1 \cos(\varphi_1)} \quad (4)$$

Here $r_{01p/s}$ are the Fresnel reflection coefficients at the ambient/layer boundary 0/1, $R_{1xp/s}$ are the generalized Fresnel amplitude reflection coefficients at the layer/substrate boundary, λ is the wavelength and i is the imaginary unit. The angle of incidence φ_0 and the angle of refraction φ_1 are connected by Snell's law: $n_0 \sin(\varphi_0) = n_1 \sin(\varphi_1)$.

In the following will omit p/s subscripts where the expressions are the same for p and s polarization (e.g. $R \equiv R_{p/s}$).

Experimentally measured quantity in reflectivity measurements is the power reflection coefficient $\mathcal{R}$ which is determined from the generally complex amplitude reflection coefficient R by

$$\mathcal{R} = I_r/I_i = |R|^2 \quad (5)$$

where I_r and I_i are incidence and reflected light intensities, respectively.

3. Complex plane trajectory of amplitude reflection coefficients

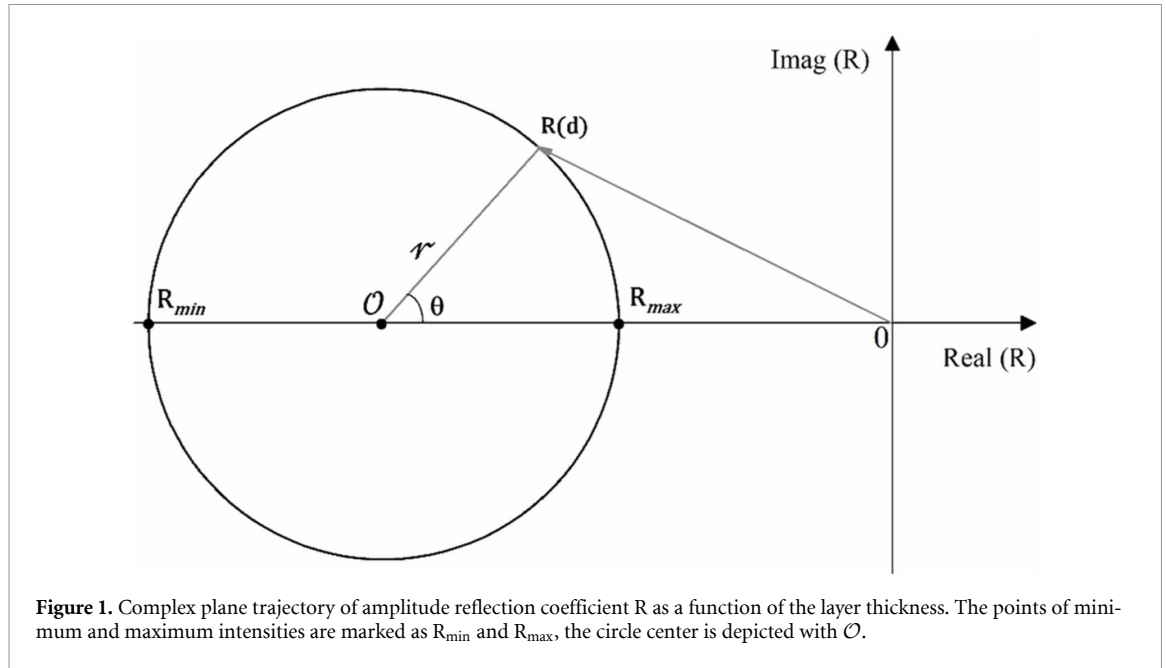
The amplitude reflection coefficients R (equation (1)) are complex functions of a complex variable Z . They are Möbius (bilinear) transformations which preserve angles and maps circles (or lines) from the Z -plane to circles (or lines) from the R -plane. When n_0 and n_1 are real, and assuming no total internal reflection at the 0/1 boundary (typically, $n_0 < n_1$, or if $n_0 > n_1$ then φ_0 is below the critical angle) the complex point Z (equation (2)) moves on the unit circle $|Z| = 1$ when the ratio $\frac{d}{\lambda}$ changes. Consequently complex-plane trajectory of $R(Z)$ is also a circle

$$R = \mathcal{O} + \tau e^{i\theta} \quad (6)$$

where θ is real (figure 1).

The center $\mathcal{O}$ and the radius τ of the circle are determined by the coefficients in the Möbius transformations of equation (1). Instead of doing straightforward but long calculations one can deduce very simply the center and the radius by observing that equation (1) is de-facto Möbius transformations of a unit circle with all real coefficients (r_{01} , $|R_{1x}|$, 1 , $r_{01}|R_{1x}|$). This means that both R and its complex conjugation R^* are points on the circle (equation (6)). Consequently, the center lies on the real axes and $\mathcal{O}$ is real. The circle intersects real axis at two extreme points

$$R_{\max} = \max(\text{real}(R)) = \mathcal{O} + \tau \quad (7a)$$



$$R_{\min} = \min(\operatorname{real}(R)) = \mathcal{O} - \tau. \quad (7b)$$

From equation (1) these two points are

$$R_{\max} = \frac{r_{01} + |R_{1x}|}{1 + r_{01}|R_{1x}|}; R_{\min} = \frac{r_{01} - |R_{1x}|}{1 - r_{01}|R_{1x}|} \text{ if } r_{01} > 0 \quad (8a)$$

$$R_{\max} = \frac{r_{01} - |R_{1x}|}{1 - r_{01}|R_{1x}|}; R_{\min} = \frac{r_{01} + |R_{1x}|}{1 + r_{01}|R_{1x}|} \text{ if } r_{01} < 0. \quad (8b)$$

From equation (7)

$$\mathcal{O} = (R_{\max} + R_{\min})/2 \quad (9)$$

which along with equation (8) gives for the circle center

$$\mathcal{O} = r_{01} \frac{1 - |R_{1x}|^2}{1 - r_{01}^2 |R_{1x}|^2}. \quad (10)$$

It is more convenient to use the amplitude reflection coefficient R_{0x} for the bare substrate in ambient with refractive index n_0 , rather than R_{1x} which is for an ambient with refractive index. In the absence of the layer ($d = 0$) we can put $Z = 1$ in equation (1) to find

$$R_{0x} = \frac{r_{01} + R_{1x}}{1 + r_{01}R_{1x}}. \quad (11)$$

Now we can express R_{1x} through R_{0x}

$$R_{1x} = \frac{r_{01} - R_{0x}}{r_{01}R_{0x} - 1} \quad (12)$$

and substitute it into equation (10). After simplification it gives the final expression for the circle radius

$$\mathcal{O} = \frac{1 - |R_{0x}|^2}{w - (R_{0x} + R_{0x}^*)} \quad (13)$$

where

$$w = r_{01} + 1/r_{01}. \quad (14)$$

For determination of the circle radius one can use similar approach using that

$$\tau = (R_{\max} - R_{\min})/2. \quad (15)$$

But the fastest way to do this is to use the fact that R_{0x} also is a point on the circle (6) thus

$$z = |\mathcal{O} - R_{0x}| . \quad (16)$$

From here

$$z = \left| \frac{wR_{0x} - (1 + R_{0x}^2)}{w - (R_{0x} + R_{0x}^*)} \right| . \quad (17)$$

4. Extracting refractive index

Using equations (13) and (16) one can verify that

$$\mathcal{O}^2 - z^2 = \mathcal{O} \cdot w - 1 . \quad (18)$$

But $\mathcal{O}^2 - z^2 = (\mathcal{O} + z)(\mathcal{O} - z) = R_{\min}R_{\max}$ and from equations (9) and (18)

$$w = r_{01} + \frac{1}{r_{01}} = 2 \frac{R_{\min}R_{\max} + 1}{R_{\min} + R_{\max}} . \quad (19)$$

This solves the inverse problem since finding w determines r_{01} and this through equation (3) or (4) (depending on the polarization) allows to find the layer refractive index n_1 .

Below we elaborate this solution for different specific cases of p/s polarizations and normal incidence

4.1. S-polarization

From equation (4)

$$w_s = r_{01s} + \frac{1}{r_{01s}} = 2 \frac{\cos 2\varphi_0 + \varepsilon}{1 - \varepsilon} \quad (20)$$

$$\varepsilon = n_1^2 / n_0^2 \quad (21)$$

and

$$\varepsilon = \frac{w_s - 2 \cos 2\varphi_0}{w_s + 2} . \quad (22)$$

Substituting w_s from equation (19) into equation (22) gives

$$n_1 = n_0 \sqrt{\frac{R_{\min,s}R_{\max,s} - (R_{\min,s} + R_{\max,s}) \cos 2\varphi_0 + 1}{(R_{\min,s} + 1)(R_{\max,s} + 1)}} . \quad (23)$$

4.2. P-polarization

From equation (3)

$$w_p = 2 \frac{(\varepsilon^2 + 1) \cos(2\varphi_0) + (\varepsilon + 1)^2 - 2}{(\varepsilon^2 - 1) \cos(2\varphi_0) + (\varepsilon - 1)^2} \quad (24)$$

which leads to quadratic equation for ε

$$(2 - w_p) \cos^2(\varphi_0) \varepsilon^2 + (2 + w_p) \varepsilon - (2 + w_p) \sin^2 \varphi_0 = 0 \quad (25)$$

with solutions

$$\varepsilon = \frac{n_1^2}{n_0^2} = \frac{-(2 + w_p) \pm \sqrt{(2 + w_p)^2 + 4(4 - w_p^2) \sin^2 \varphi_0 \cos^2 \varphi_0}}{2(2 - w_p) \cos^2 \varphi_0} \quad (26)$$

where $w_p = 2 \frac{R_{\min,p}R_{\max,p} + 1}{R_{\min,p} + R_{\max,p}}$ can be found from equation (19) and substituted in equation (26).

Equations (23) and (26) show that the proposed method does not impose a thickness range through the analytical inversion itself. Instead, the practical thickness limits are determined by the experimental requirement that at least one interference cycle to be observable and adequately resolved. This is discussed in more detail in section 8.

4.3. Normal incidence

At normal incidence (most encountered case in practice) $\varphi_0 = 0$ and p/s polarizations are indistinguishable, and equation (23) or (26) give for the refractive index simple expression

$$n_1 = n_0 \sqrt{\frac{(R_{\min}-1)(R_{\max}-1)}{(R_{\min}+1)(R_{\max}+1)}}. \quad (27)$$

4.4. Simple nonabsorbing substrate

Interestingly, for a transparent, homogeneous substrate with a real refractive index n_2 both refractive indexes n_1 and n_2 can be found solely from $R_{\min}$ and $R_{\max}$. In this case $R_{0x} = r_{02}$ where r_{02} are Fresnel coefficients defined by equations (3) and (4) by changing index 1 with 2. Equation (13) becomes

$$\mathcal{O} = \frac{1 - r_{02}^2}{w - 2r_{02}}. \quad (28)$$

Substituting above in equation (13) $\mathcal{O}$ from equation (9) and w from equation (19) one obtains quadratic equation for r_{02}

$$r_{02}^2 - (R_{\min} + R_{\max})r_{02} + R_{\min}R_{\max} = 0 \quad (29)$$

which has two solutions

$$r_{02} = \{R_{\min}, R_{\max}\}. \quad (30)$$

This result follows directly from geometric considerations. Since $R_{0x} = r_{02}$ must lie on the circle defined by equation (6) and is real, it must align with one of the only two real points on the circle.

It can be seen by solving for n_2 Fresnel equations $r_{02p/s}$ that

$$\frac{n_2^2}{n_0^2} = \sin^2 \varphi_0 + \cos^2 \varphi_0 \frac{(1 - r_{02s})^2}{(1 + r_{02s})^2} \quad (31)$$

$$\frac{n_2^2}{n_0^2} = \frac{1 \pm \sqrt{1 - 4\sin^2 \varphi_0 \cos^2 \varphi_0 \frac{(1 - r_{02p})^2}{(1 + r_{02p})^2}}}{2\cos^2 \varphi_0 \frac{(1 - r_{02p})^2}{(1 + r_{02p})^2}}. \quad (32)$$

Which along with equation (30) gives possible solutions for the substrate refractive index. At normal incidence equations (31) and (32) simplify to

$$n_2 = n_0 \left| \frac{1 - r_{02}}{1 + r_{02}} \right|. \quad (33)$$

5. Multiplicity of solutions

The refractive index is determined solely by the extreme values of the real part of the amplitude reflection coefficients. At these points, the reflection coefficients are real, and the phase can only be 0 or π . Thus, $R_{\min}$ and $R_{\max}$ are determined from measured extreme values of power reflection coefficients (5) up to a sign:

$$R_{\min, \max} = \pm \sqrt{\mathcal{R}_{\min, \max}}. \quad (34)$$

This yields four possible sign combinations: $(++)$, $(-+)$, $(+-)$ and $(--)$, resulting in four mathematical solutions for s-polarization from equation (23). For p-polarization, equation (26) gives eight solutions from which complex (always in complex conjugated pairs) must be rejected.

At grazing incidence, only one solution from each polarization coincides, representing the true refractive index. Thus, simultaneous p- and s-polarization measurements at grazing incidence provide an unambiguous refractive index determination.

At normal incidence equation (27) gives again four solutions, but they are pairwise reciprocal $n_{1++} = \frac{1}{n_{1--}}$ and $n_{1+-} = \frac{1}{n_{1-+}}$. In the typical case where $n_1 > n_0$, only the two solutions greater than 1 are valid choices.

Table 1 illustrates the application of the above approach using simulated reflectivity data for multilayer system which consists of a top layer (refractive index $n_1 = 1.5982$) deposited onto a complex

Table 1. Calculated top layer refractive index from simulated data for a four-phase system with an ambient refractive index $n_0 = 1$, a top layer with refractive index $n_1 = 1.5982$, a 100 nm middle SiO₂ layer with refractive index 1.46, and a silicon substrate with refractive index $3.88 - 0.019i$. The angle of incidence is $\varphi_0 = 60^\circ$ or $\varphi_0 = 0^\circ$, and the wavelength $\lambda = 632.8$ nm. The unique refractive index solution obtained from combined p- and s-reflectivity data is highlighted in bold.

	$R_{\min,\max} = \pm \sqrt{\mathcal{R}_{\min,\max}}$	++	+-	-+	--
p-polarization, $\varphi_0 = 60^\circ$	$R_{\min}$	0.3668	-0.3668	-0.4097	-0.4097
	$R_{\max}$	0.4097	0.4097	0.3668	-0.3668
	n_1 (equation (26))	0.8829	0.9782	1.5982	$0.7586 + 0.4329i$
		4.4543	1.8622	1.0304	$0.7586 - 0.4329i$
s-polarization, $\varphi_0 = 60^\circ$	$R_{\min}$	0.1382	-0.1382	-0.8101	-0.8101
	$R_{\max}$	0.8101	0.8101	0.1382	-0.1382
	n_1 (equation (23))	0.8774	0.8858	1.5982	1.9742
Normal incidence, $\varphi_0 = 0^\circ$	$R_{\min}$	0.2898	-0.2898	-0.6453	-0.6453
	$R_{\max}$	0.6453	0.6453	0.2898	-0.2898
	n_1 (equation (27))	0.3445	0.6257	1.5982	2.9026

substrate—SiO₂/Si. The thickness of the oxide layer is 100 nm and its refractive index and those of the substrates are 1.46 and $3.88 - 0.019i$ respectively.

Multiple mathematical solutions of the inverse task is a common situation for layer parameter determination. For example, finding layer RI and thickness from ellipsometry data leads to five numerical solutions [16, 17].

For some systems, due to experimental uncertainties, it is possible to obtain two solutions which are physically plausible and numerically close. In such cases, several strategies can be employed to select the real physical solution:

1. *Measurements at a second incidence angle.* As discussed in section 6.3, combining measurements taken at two incidence angles substantially reduces the ambiguity because the correct solution must satisfy both datasets simultaneously.
2. *Simultaneous p- and s-polarization measurements.* As shown in section 5, the physically correct solution is the one common to both polarizations, particularly at larger incidence angles.
3. *Prior physical information.* Even a broad estimate of the expected refractive-index range can eliminate many candidate solutions. For example, most transparent organic liquids lie in the range $1.3 < n < 1.6$, while common oxides typically lie between 1.4 and 2.5.
4. *Consistency across multiple extrema pairs.* For spectroscopic data, the correct branch should produce a smooth refractive-index dispersion curve, whereas incorrect branches typically exhibit discontinuities or nonphysical wavelength dependence.

These procedures can readily be implemented algorithmically. For example, an automated routine could evaluate all mathematical solutions and select the branch that minimizes the variation between measurements obtained at different angles or polarizations, or alternatively the branch producing the smoothest dispersion curve.

6. Calibration

Absolute scaling is a critical factor in optical measurements—particularly in reflectivity and interference methods—where the analysis relies on absolute values of reflectance or transmittance (as in the method proposed above). The dominant source of error in determining power reflectivity coefficients is the calibration of measured intensities: experimental setups typically provide relative intensity variations with high accuracy, but obtaining the absolute ratio of reflected to incident power is considerably more difficult.

To account for this, the measured reflectance intensity I is related to the true reflectivity $\mathcal{R}$ through a calibration factor that depends on the light source output, detector responsivity, and overall optical throughput, all of which are wavelength dependent:

$$\mathcal{R} = c^2 I \quad (35)$$

(The squared form of c is introduced for later convenience.) Consequently, the amplitude reflection coefficients obtained from equation (5) must be rescaled everywhere to cR .

Various methods have been proposed for determining the scaling factor [8, 18, 19], with calibrated reflectance standards being the most commonly used approach [20]. Here we propose a calibration procedure closely aligned with the measurement method, fast, simple, and especially suited for camera-based measurements.

If the detector response is nonlinear, a single calibration coefficient is insufficient, and systematic errors may be introduced into the extracted extrema values.

6.1. Calibration using a homogeneous nonabsorbing substrate

Consider the case of interference fringes produced by a dielectric film of refractive index n_1 deposited on a simple, homogeneous substrate with real refractive index n_2 —for example an oil layer on glass. The amplitude reflection coefficient of the bare substrate, r_{02} , is then purely real (see equations (3) and (4) with index 1 replaced by 2). Because r_{02} must lie on the Mobius-transformed circle described by equation (6), it must coincide with one of the two real extrema of that circle: $R_{\max}$ (for $n_1 < n_2$) or $R_{\min}$ (for $n_1 > n_2$).

From this, the calibration coefficient follows directly as

$$c = |r_{02}/R_{\min/\max}|. \quad (36)$$

Note that for normal incidence this simplifies to $c = \left| \frac{n_0 - n_2}{(n_0 + n_2)R_{\min/\max}} \right|$.

6.2. Calibration using two different substrates

Consider now the reflection extrema of the same transparent layer, measured at the same incidence angle, but deposited on two different arbitrary substrates. Using s-polarization is advantageous because, as shown below, it yields a unique solution for the calibration coefficient and simultaneously determines the layer refractive index.

Since equation (23) must return the same refractive index n_1 for both substrates, the extrema measured in the two systems must satisfy this condition.

For brevity, let the pair of extrema $(R_{\min,s}, R_{\max,s})$ from the first substrate be denoted as (A, B) , and those from the second substrate as (A', B') . Introducing the calibration factor c , equation (23) leads to the requirement

$$\frac{(Ac-1)(Bc-1)+c(A+B)(1-\cos 2\varphi_0)}{(Ac+1)(Bc+1)} = \frac{(A'c-1)(B'c-1)+c(A'+B')(1-\cos 2\varphi_0)}{(A'c+1)(B'c+1)}. \quad (37)$$

This equation contains a single unknown—the calibration multiplier c . Although the resulting polynomial is of fourth degree, it factorizes nicely into

$$((AB(A'+B') - A'B'(A+B))c^2 - A - B + A' + B')c(1 + \cos 2\varphi_0) = 0. \quad (38)$$

After discarding the trivial root $c = 0$ and noting that $1 + \cos 2\varphi_0 \neq 0$ for any non-grazing incidence, the calibration factor is uniquely determined by

$$c^2 = \frac{A+B-A'-B'}{AB(A'+B')-A'B'(A+B)}. \quad (39)$$

Remarkably, this expression is *independent* of the incidence angle φ_0 and thus remains valid for *any* angle. This is a major practical advantage of this calibration method: it does not require knowledge of the exact incidence angle, thus avoiding error propagation associated with its experimental determination.

Once c is determined, all measured intensities can be rescaled according to equation (35), after which the layer refractive index can be obtained from any single pair of corrected extrema using equation (23).

6.3. Calibration using two different incidence angles

If fringe extrema are measured at two different incidence angles, the calibration factor c and the layer refractive index n_1 can be determined simultaneously. One of the angles must be non-normal. Either polarization can be used, though s-polarization at both angles provides the simplest and most robust formulation and below we consider this case.

For s-polarization at angles φ_{01} and φ_{02} , let (A_1, B_1) and (A_2, B_2) denote the corresponding reflectivity extrema (min, max). Since each pair must yield the same refractive index via equation (23), the calibration factor is obtained as a root of the quadratic equation $a_2 c^2 + a_1 c + a_0 = 0$ with coefficients

$$a_2 = A_1 B_1 (B_2 + A_2) (\cos 2\varphi_{02} + 1) - A_2 (B_1 + A_1) B_2 (\cos 2\varphi_{01} + 1)$$

$$a_1 = (B_1 + A_1) (B_2 + A_2) (\cos 2\varphi_{02} - \cos 2\varphi_{01})$$

$$a_0 = (B_2 + A_2) (\cos 2\varphi_{02} + 1) - (B_1 + A_1) (\cos 2\varphi_{01} + 1).$$

After determining c , the measured intensities are rescaled using equation (35), and the layer refractive index can then be obtained from either pair of corrected extrema using equation (23).

The drawback of using two incidence angles, compared with the methods in the previous sections, is that it yields two possible values for the calibration factor. As a result, the number of refractive-index solutions, among which the physically correct one must be selected, is doubled.

7. Error propagation analysis

To assess the robustness of the proposed method, we performed a first-order sensitivity analysis. For example, for the most common case of normal incidence, differentiation of equation (27) yields

$$\frac{\delta n_1}{n_1} = \frac{\delta R_{\min}}{1 - R_{\min}^2} + \frac{\delta R_{\max}}{1 - R_{\max}^2}.$$

The corresponding standard uncertainty is

$$\sigma_{n_1} = n_1 \sqrt{\frac{\sigma_{R_{\min}}^2}{(1 - R_{\min}^2)^2} + \frac{\sigma_{R_{\max}}^2}{(1 - R_{\max}^2)^2}}$$

Figure 2 (left) illustrates this uncertainty propagation graphically in the parameter space of the extreme values of power reflection coefficients ($R_{\min}, R_{\max}$). The shaded region represents a $\pm 1\%$ uncertainty in the measured extrema around the simulated values from table 1. The intersections of this region with the constant refractive-index contours determine the resulting uncertainty intervals in n_1 .

For the simulated example of table 1 ($R_{\min} = -0.6453$, $R_{\max} = 0.2898$), a 1% uncertainty in both extrema results in $\delta n_1 \approx 0.02$, corresponding to a relative error of approximately 1.4%. The analysis shows that the method is generally robust with respect to measurement noise.

Increased sensitivity occurs when the extrema approach perfect reflection ($R_{\min} \rightarrow 1$ or $R_{\max} \rightarrow 1$) (see figure 2—right), where the contour spacing becomes smaller and uncertainty amplification becomes more pronounced. In typical thin-film interference experiments, where the extrema remain sufficiently less than unity, the resulting uncertainty remains moderate. Similar behavior is obtained for the more general case of oblique incidence.

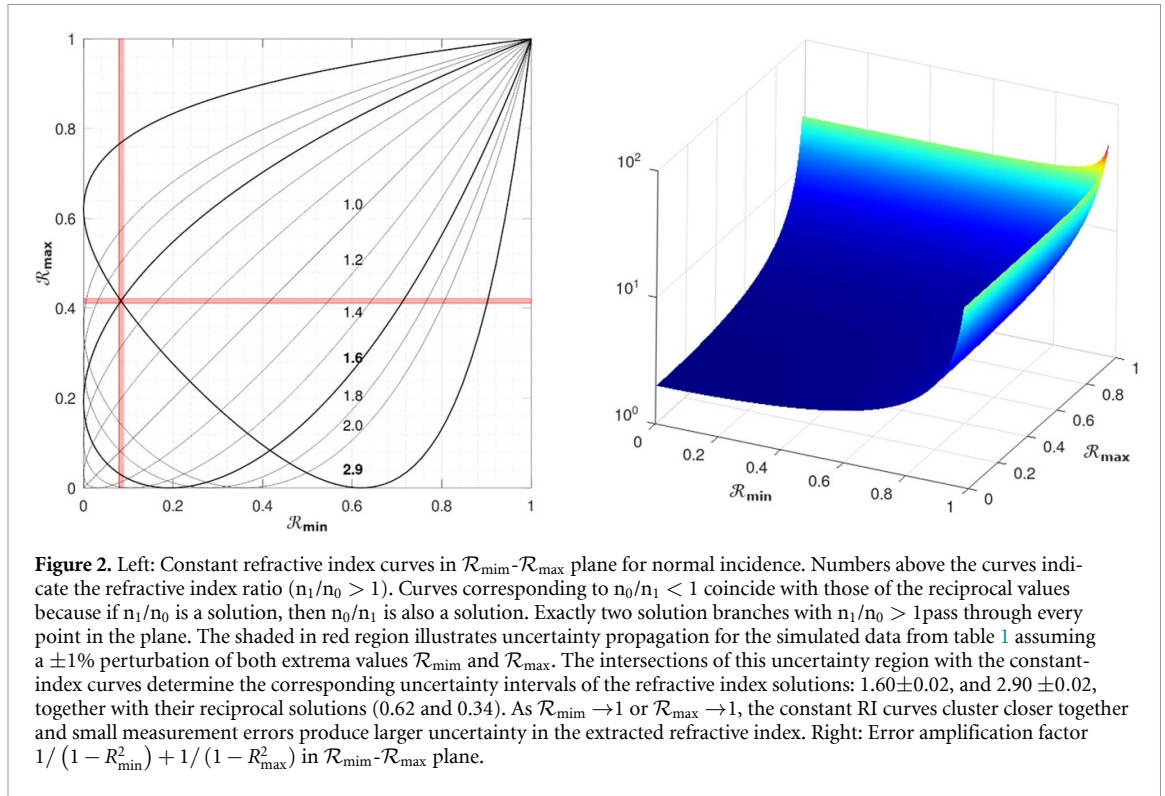
8. Limitations and applicability conditions

8.1. Surface roughness

The effect of roughness depends on its characteristic length scale relative to the wavelength. For optically smooth surfaces ($\sigma \ll \lambda$, where σ is the RMS roughness), roughness primarily reduces the coherent specular reflection amplitude which exponentially decreases with $(\sigma/\lambda)^2$.

If small, this systematic error in reflectance will have the same influence as discussed in error propagation analysis above.

If the roughness in the reference and sample is the same (as in the case of using bare surface with known reflectance for reference and the same surface for layer spreading), both the minimum and maximum reflected intensities are reduced by nearly the same multiplicative factor. In this case the calibration coefficient c remains unchanged in the first order approximation and does not affect the extracted refractive index.



8.2. Low-contrast case

The interference fringe contrast C (visibility) is given by:

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{\mathcal{R}_{\max} - \mathcal{R}_{\min}}{\mathcal{R}_{\max} + \mathcal{R}_{\min}}.$$

At low contrast $C \ll 1$ and $\mathcal{R}_{\max} \sim \mathcal{R}_{\min}$. For example, this can happen when $n_1 \approx n_0$, or when $n_1 \approx n_{\text{substrate}}$ (for a simple substrate). In the extreme case of equality, no fringes are observable ($\mathcal{R}_{\max} = \mathcal{R}_{\min}$). From the theoretical standpoint, the proposed method remains valid even in the low-contrast limit. The refractive-index extraction is based on the extrema values themselves rather than on fringe spacing or contrast, and equation (19) remains well defined. When $I_{\max} - I_{\min}$ approaches the intensity noise level δI of the measurement system, extrema minimum and maximum become equal, and there is uniform reflectivity independent of the layer thickness. Provided that the region covers sufficiently large thickness change (see section 8.3), then $\mathcal{R}_{\max} \approx \mathcal{R}_{\min}$, but they are determined with the same relative error $\delta I/I$.

In most fringe-based techniques, decreasing contrast is expected to degrade the accuracy of the extracted parameters. Here, however, the refractive index depends on the product and the sum of the extrema, but not on their difference. Provided the absolute intensities are determined with sufficient fidelity, a reduction in fringe contrast does not, by itself, lead to a larger error in the retrieved refractive index—an unusual but valuable feature of the method.

A simple example is a glass substrate with refractive index $n_2 = 1.51$ and a top layer with the same refractive index $n_1 = n_2$. At any layer thickness it will be fulfilled that $\mathcal{R}_{\max} = \mathcal{R}_{\min} = 0.041$ and therefore the contrast is zero. Still equation (27) works. It gives four solutions: -0.6623 , 1.0000 , 1.0000 and 1.5100 , among which are the layer and the ambient refractive indexes, without any prior knowledge of the system. This is also evident in figure 2 (right), where the zero contrast corresponds to the line $\mathcal{R}_{\min} = \mathcal{R}_{\max}$ (which always gives $n_1/n_0 = 1$ as one of the solutions). However, zero contrast does not introduce a large error, as the inversion remains well-behaved near that limit.

8.3. Lower and upper thickness limits

The method does not require knowledge of the absolute film thickness. However, it does require that the available data contain at least one complete interference cycle—i.e. one reflectivity minimum and one adjacent maximum—so that both $\mathcal{R}_{\min}$ and $\mathcal{R}_{\max}$ can be identified. The constraints for fulfilling this condition differ depending on the measurement mode:

- In variable-thickness mode (wedge films, droplets, growth, etching, oxidation—variable thickness, fixed wavelength), the requirement is a sufficient change in thickness to span one full fringe period. Upper limit is imposed due to the finite coherence length of the source.
- In spectroscopic mode (fixed thickness, wavelength scan), the requirement is a sufficient spectral range to include at least one adjacent min–max pair. Additionally, as the film thickness increases, the spectral spacing between fringes decreases, imposing an upper thickness limit set by the spectrometer resolution.

The following subsections define these practical bounds.

8.3.1. Lower thickness limit

For variable-thickness mode the lower bound is the minimum thickness variation Δd that produces sufficient optical phase difference for observation of at least one minimum-maximum pair:

$$\Delta\Phi = \frac{4\pi n_1 \Delta d \cos\varphi_1}{\lambda} \geq 2\pi.$$

Thus, the minimum thickness range required to observe one full interference period is approximately

$$\Delta d \approx \frac{\lambda}{2n_1 \cos\varphi_1}.$$

For spectroscopic measurements at fixed thickness, the condition for observing at least one min–max pair within the available wavelength range $[\lambda_1, \lambda_2]$ is

$$\frac{4\pi n_1 d}{\lambda_1} - \frac{4\pi n_1 d}{\lambda_2} \geq 2\pi,$$

which, assuming constant n_1 , gives a lower bound on the absolute thickness: $d_{\min} \geq \frac{\lambda_1 \lambda_2}{2n_1(\lambda_2 - \lambda_1)}$.

For example, with $\lambda = 400$ – 800 nm, $n_1 = 1.5$, this gives $d_{\min} \approx 267$ nm. For thinner films, no extremum pair appears within the measured spectral window.

8.3.2. Upper thickness limit

There is no fundamental upper thickness limit in the analytical formulation itself, since the refractive index is determined solely from the reflectivity extrema and the layer thickness does not explicitly enter the inversion equations.

In practice, however, there are measurement-dependent upper thickness limits that differ for spectroscopic and thickness-dependent implementations of the method.

For spectroscopic measurements, increasing thickness decreases the spectral spacing between adjacent fringes,

$$\Delta\lambda \approx \frac{\lambda^2}{2n_1 d}.$$

When the fringe spacing becomes comparable to the spectrometer resolution $\delta\lambda$, neighboring extrema can no longer be resolved reliably. Therefore, the upper thickness limit is

$$d_{\max} \sim \frac{\lambda^2}{2n_1 \delta\lambda}.$$

For example, at $\lambda = 550$ nm, $n_1 = 1.5$, and a spectral resolution of 1 nm, the maximum thickness for reliable fringe resolution is approximately $100 \mu\text{m}$.

In variable thickness mode, this upper limitation is generally absent because the extrema are separated spatially rather than spectrally. But in this case another limitation is imposed by the finite coherence length of the illumination. For large thicknesses, averaging over the finite spectral bandwidth reduces the modulation depth of the interference pattern, resulting in measured minima that become higher and maxima that become lower than their ideal values. For a source with central wavelength λ and spectral width $\Delta\lambda$, the coherence length is approximately

$$L_c \approx \frac{\lambda^2}{\Delta\lambda}.$$

Interference visibility starts to deteriorate when $2n_1 d \cos \varphi_1 \gtrsim L_c$.

This gives a practical upper thickness

$$d_{\max} \approx \frac{\lambda^2}{2n_1 \Delta \lambda \cos \varphi_1}.$$

For our experimental conditions: normal incidence, $\lambda = 550 \text{ nm}$, $\Delta \lambda = 7 \text{ nm}$, $n_1 = 1.5$, which gives $L_c \approx 43 \mu\text{m}$ and $d_{\max} \approx 14 \mu\text{m}$.

Beyond that, the fringe contrast will be substantially small because of finite coherence.

This is the reason that the extrema used in the present work are selected from the first well-resolved fringes. Higher-order fringes experience reduced visibility due to the finite coherence length of the illumination source, leading to larger uncertainty in extrema determination.

9. Experimental validation—homogeneous substrates

9.1. Materials and methods

Several non-volatile liquids were used to test the proposed method. They include dodecane (purity $>99.0\%$ (GC)—TCI Chemicals), triolein (Chemica (99%), CAS: 122-32-7), silicone oil (WACKER® AK 100) and liquid paraffin (Ph. Eur. [0240], Chemax pharma, Bulgaria).

Different substrates with high contrast in reflection coefficients were chosen: a 10 mm thick glass slide (Schott); optical glass with a 96 nm top layer of TiO_2 , e-beam deposited; a 100 nm Al layer e-beam deposited onto a glass coverslip. Prior the experiments the substrates were cleaned with isopropyl alcohol in ultrasonic bath for 120 s, rinsed with plenty of deionized water (with specific resistivity of 18.2 MΩ cm) and dried under nitrogen stream.

The experimental procedure was as follows. A microliter drop of the test liquid was deposited onto the substrate by using a micropipette. The liquid spread on the substrate forming a thin lens-like sheet with variable thickness. The resulting interference image is due to the layer thickness-dependent reflectivity.

Optical microscope Jena Tech, working in reflection mode was used for the thin layer observation. Digital camera Lumenera Infinity 2 was used for image capture. The automatic adjustments of the settings were turned off, and the capture quality was set to 16-bit, which ensures 0–65 535 dynamic range per pixel. A 550 nm interference filter with 7 nm halfwidth was put just after the light source to ensure a monochromatic illumination. The free software Image J [21] was used for analyzing the acquired images. The images are captured in TIFF format. Prior to any image processing, the color channels were split and green was used.

An UV VIS NIR spectrometer AvaSpec-2048TEC-2, Avantes, combined with halogen and deuterium light sources, was used for spectral measurements in the range from 350 nm to 1100 nm. An optical standard BK7 glass (thickness 10 mm) with known refractive index dispersion [22] was used as a reference sample. The spectrometer was operated in reflection mode using a reflection probe optical fiber.

9.2. Dielectric layer on homogeneous nonabsorbing substrate

In the case of a simple substrate with a known refractive index, its reflection can be used for calibration if the captured intensity. For this purpose, a specially developed Image J plugin was used to find the minimum and the maximum intensities of the reflected light.

An example is shown in figure 3 for the case of dodecane on glass. The maximum fringe intensity corresponds to the reflectance of the bare substrate. Using the known glass refractive index (1.5187), the calibration coefficient is determined as described above (section 6.1). The normalized reflection extrema are then used to extract the refractive index of the corresponding liquid.

To account for the noise of the images, the refractive index was determined over 10 different areas near the edge of the liquid. Computed data is shown in table 2 along with the standard deviation. The third column represents literature data for the value of the refractive index.

9.3. Dielectric layer on two different substrates

Measuring the minimum and maximum reflection intensities of two different substrates allows us to extract the calibration coefficient directly, without any knowledge of their parameters like refractive indexes and thicknesses. For demonstration the proposed approach, the refractive index of dodecane is extracted from the reflection coefficients of two different substrates—Al layer deposited on top of a glass coverslip and TiO_2 layer deposited onto an optical glass. Since the calibration coefficient (equation (39)), and hence the stability of the solution, is determined by the differences in maximum and minimum

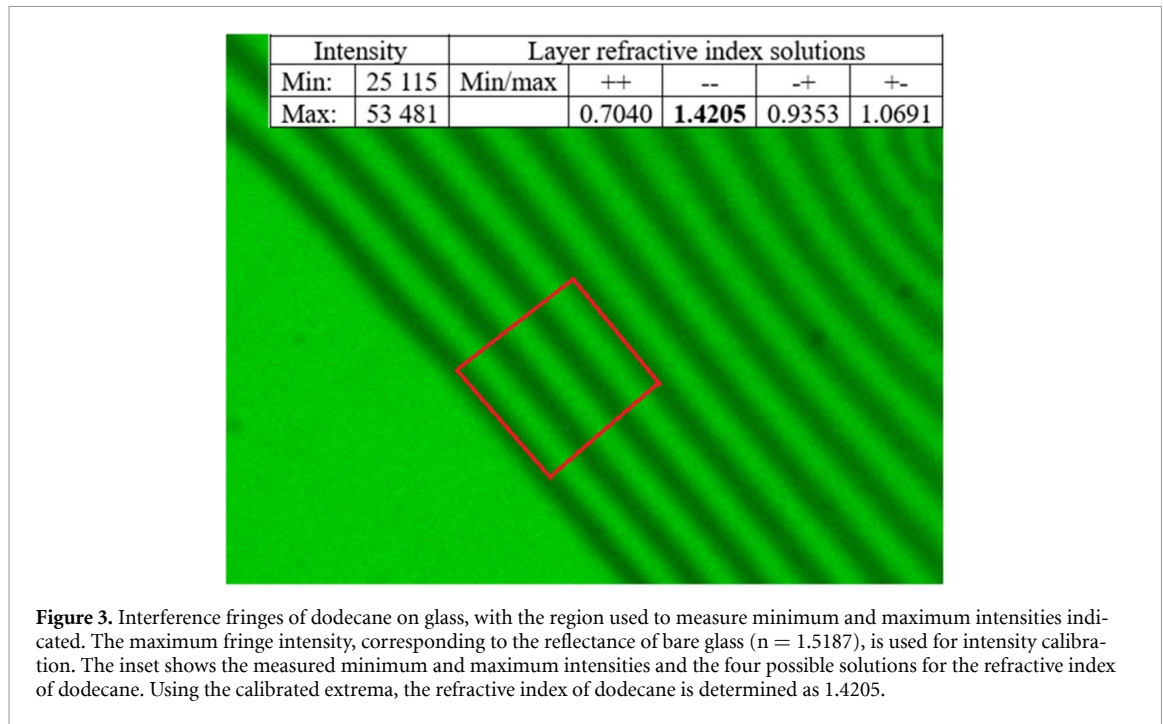


Figure 3. Interference fringes of dodecane on glass, with the region used to measure minimum and maximum intensities indicated. The maximum fringe intensity, corresponding to the reflectance of bare glass ($n = 1.5187$), is used for intensity calibration. The inset shows the measured minimum and maximum intensities and the four possible solutions for the refractive index of dodecane. Using the calibrated extrema, the refractive index of dodecane is determined as 1.4205.

Table 2. Calculated refractive index and standard deviation of four liquids, deposited onto a glass substrate. The fourth column contains data taken from literature.

Liquid	Computed RI	Standard deviation	Literature data	References
Dodecane	1.424	0.002	1.4210	[23]
Triolein	1.461	0.002	1.4676	[23]
Silicone oil	1.418	0.003	1.403	[24]
Liquid paraffin	1.460	0.003	1.461	^a

^a The refractive index of liquid paraffin was measured using Abbe refractometer.

intensities between the two substrates, they are selected so that this value is a maximum—a dielectric structure and a metal. This also requires adjustment of the camera settings when capturing the images. As a preliminary experiment, the linearity of the camera intensity as a function of the gain was checked and confirmed, at a fixed exposure time. This allows images to be captured at different gain values, after which the intensity is recalculated.

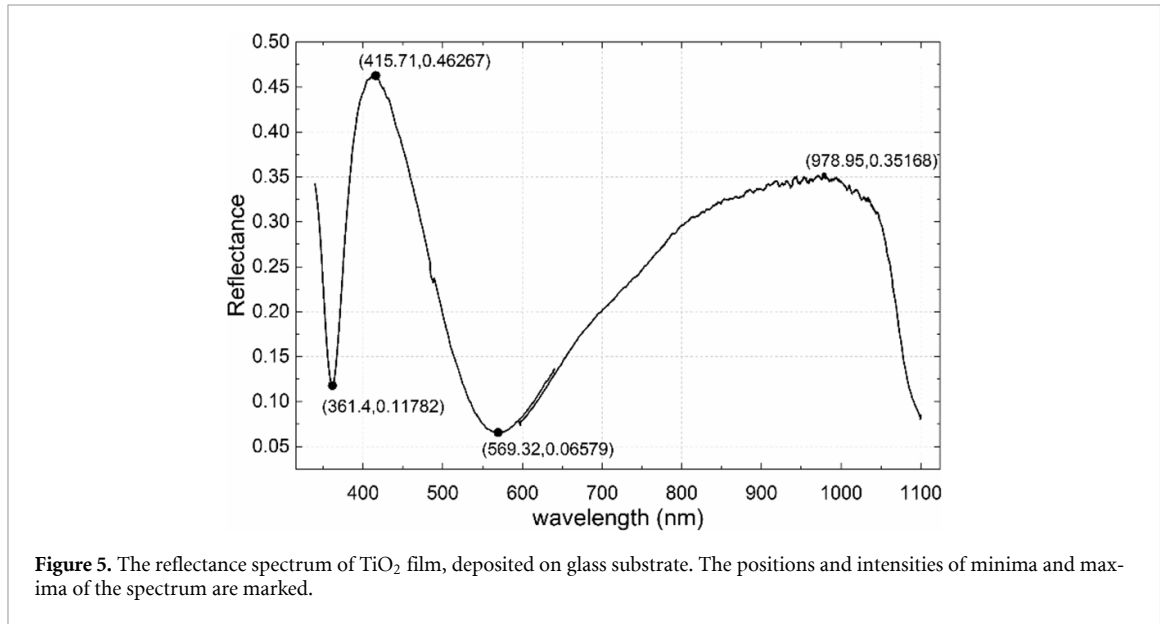
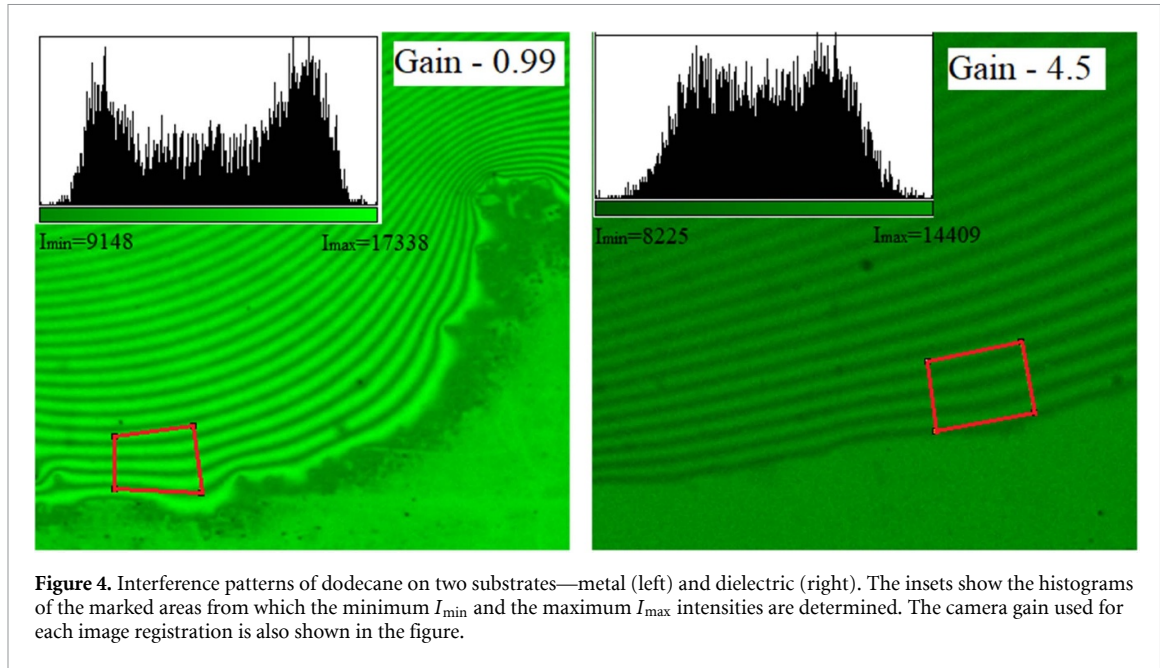
Figure 4 shows two interference patterns of dodecane. The left one is for metal surface and camera gain 0.99, while the right is for dielectric substrate with image taken at gain of 4.5. The minimum and maximum intensities of the selected areas are computed using the histogram function of Image J.

The stability of the solution was tested by introducing a random noise in the measured intensities in the order of one standard deviation. The computed refractive index of dodecane is 1.415 ± 0.02 .

9.4. Spectroscopic data

Here we demonstrate the applicability of the new analytical solution also in treating spectroscopic reflection data. Unlike the thickness-dependent approach where interference fringes arise from varying optical path length at fixed wavelength, spectroscopic measurements produce fringes through wavelength variation at fixed film thickness. For each adjacent minimum-maximum pair, equation (27) yields the refractive index at the mean wavelength, effectively sampling the material's dispersion curve.

The sample used in the spectroscopic experiments was a TiO_2 layer, deposited onto a glass substrate. In that case, the calibration coefficient as a function of the wavelength was extracted from the reflectance spectra of a reference sample—BK7 optical glass. Figure 5 shows the calculated reflection of the TiO_2 sample as a function of the wavelength, taken at normal incidence. The two separate spectra correspond to the two detection channels of the spectrometer.



If the refractive index dispersion is sufficiently weak over the interval $\Delta\lambda = |\lambda_{\max} - \lambda_{\min}|$ we can expand the refractive index around the midpoint wavelength $\bar{\lambda}$

$$n(\lambda) = n(\bar{\lambda}) + \frac{dn}{d\lambda}\Delta\lambda + \dots$$

Consequently, the validity of the spectroscopic approximation is controlled by the dimensionless quantity

$$\eta = \left| \frac{dn}{d\lambda} \right| \Delta\lambda.$$

When $\eta \ll 1$, the refractive index changes only slightly between adjacent extrema and the midpoint approximation is justified. When η approaches a few percent, the systematic deviations become noticeable. Thus, the method is most accurate when either the dispersion is weak or the spacing between the adjacent extrema is small.

For the TiO_2 sample studied here, the refractive index varies smoothly over the visible range, and the extrema spacing is sufficiently small, resulting in good agreement with the literature data (figure 6).

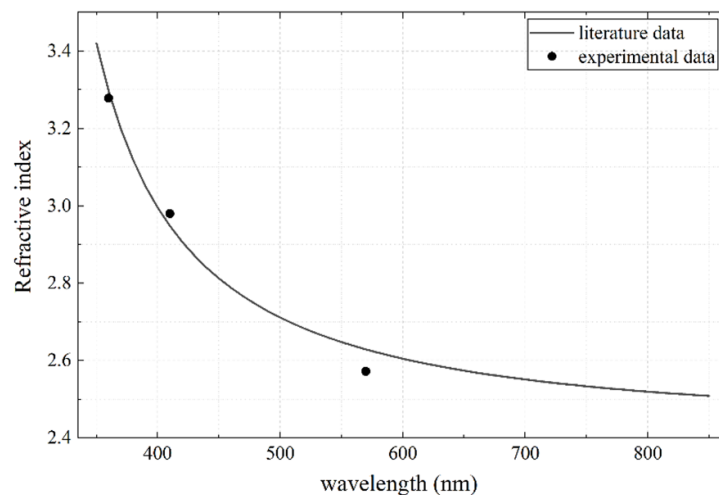


Figure 6. The refractive index of TiO₂ as a function of the wavelength. Dots—computed values using equation (27); solid line—literature data, taken from [25].

However, for strongly dispersive materials, particularly near absorption edges, the averaging approximation may become inadequate. In such cases a more rigorous treatment incorporating the wavelength dependence of $n(\lambda)$ between extrema would be required.

The refractive index of the TiO₂ layer was calculated, using equation (27), from every consecutive couple of $R_{\min}/R_{\max}$. The result, which is an average value in the spectral range defined by $\lambda(R_{\min})$ and $\lambda(R_{\max})$ is shown with dots on figure 6. The refractive index dispersion of TiO₂ [25] is also shown for comparison.

10. Discussion

Unlike fringe-spacing methods, which typically require knowledge of the film thickness or multiple-angle measurements to separate thickness and refractive index effects, the proposed approach determines the refractive index solely from the extrema values of the reflectivity signal and does not require prior knowledge of the film thickness. Furthermore, the method does not require knowledge of the optical properties of the substrate and remains applicable to multilayer, absorbing, and depth-dependent substrate structures, if they satisfy the assumptions described in section 2. An additional advantage is that, provided the absolute intensities are measured reliably, the method remains accurate even at low contrast.

Compared with ellipsometry, the proposed method requires substantially simpler instrumentation and data analysis. However, ellipsometry generally provides higher accuracy and can simultaneously determine several optical parameters, including thickness and absorption coefficients. The present approach is therefore intended as a complementary method rather than a replacement for ellipsometry.

The principal advantage of the present method is not higher accuracy than other methods, but the ability to extract the refractive index from only the measured reflectivity extrema without requiring knowledge of either film thickness or substrate optical properties.

An additional advantage of the proposed method is its applicability to spatially localized samples and films with nonuniform thickness. Since the method relies only on the extrema of an interference pattern, it can be applied directly to microscopic objects such as liquid droplets, menisci, bubbles, or other naturally occurring thickness gradients observed under an optical microscope. In such systems, the thickness profile need not be known and may be highly irregular. The method exploits the naturally occurring variation in thickness to generate the required reflectivity extrema.

By contrast, conventional reflectometric and ellipsometric approaches are often optimized for laterally uniform films and typically require either a well-defined sample geometry, model-based fitting of thickness and optical constants, or a measurement spot sufficiently large and homogeneous for reliable analysis. The proposed approach can therefore provide a simple alternative for localized microscopic samples where conventional characterization techniques become difficult to implement.

This work presents a novel analytical approach for determining the refractive index of transparent thin films from reflectivity extrema. By exploiting the Möbius transformation of the amplitude reflection coefficient in the complex plane, the method establishes a direct relationship between the extrema of the reflectivity signal and the film refractive index. The formulation is applicable to both thickness-dependent and spectroscopic measurements under the assumptions stated in section 2 and remains independent of explicit thickness determination.

Experimental and simulation validation performed on several representative systems demonstrates good agreement with literature values and confirms the practical applicability of the approach. Owing to its analytical simplicity, minimal experimental requirements, and compatibility with spatially localized and nonuniform samples, the method may provide a useful complementary tool for thin-film characterization and optical microscopy applications.

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Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: <https://data.mendeley.com/datasets/dwx74xbjxj/1> [26].

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